Quiz 4, Calculus III – Calculators okay

Dr. Graham-Squire, Fall 2013

Name: Key

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1. (4 points) Use any method you want to solve the optimization problem (you can use methods from sections 13.8/13.9 or Lagrange multipliers). The product of three numbers x, y and z is ~~27~~ $\frac{32}{3}$. Find values for x, y and z such that the sum $x + 2y + 3z$ is a minimum.

$xyz = \cancel{27}\ \frac{32}{3}$ $\quad x + 2y + 3z = g(x,y,z)$ ✓

$\Rightarrow x, y,$ and z cannot equal 0.

$f(x,y,z) = xyz, \quad c = 27$

$\nabla f = \langle yz, xz, xy \rangle \qquad \nabla g = \langle 1, 2, 3 \rangle$

$\Rightarrow \lambda = yz, \quad 2\lambda = xz, \quad 3\lambda = xy$ ✓

$\Downarrow$

$2y\cancel{z} = x\cancel{z}$

$\Rightarrow 2y = x$

$y = \frac{x}{2}$ ✓

and $\lambda = yz$

$\Rightarrow 3\cancel{y}z = x\cancel{y}$

$\Rightarrow 3z = x$

$\Rightarrow z = \frac{x}{3}$

Now $xyz = \frac{32}{3}$

$\Rightarrow x\left(\frac{x}{2}\right)\left(\frac{x}{3}\right) = \frac{32}{3}$

$\frac{x^3}{6} = \frac{32}{3}$ ✓

$x^3 = \frac{32}{\cancel{3}} \cdot \cancel{6}^{\,2} = 64$

$\boxed{x = \sqrt[3]{64} = 4}$

$\Rightarrow$ $\boxed{y = 2}$

$\boxed{z = \frac{4}{3}}$

$g\left(4, 2, \frac{4}{3}\right) = 8 + 4 + 4 = 16$ is a min

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2. (3 points) Evaluate the iterated integral $\int_1^4 \int_1^{\sqrt{x}} 2ye^{-x}\, dydx$. Show your work and leave your answer in exact form.

$$= \int_1^4 \left(y^2 e^{-x} \Big|_1^{\sqrt{x}} \right) dx$$

$$= \int_1^4 (xe^{-x} - e^{-x})\, dx$$

$u = x \quad dv = e^{-x}dx$

$du = dx \quad v = -e^{-x}$

$$= \int_1^4 xe^{-x}\, dx - \int_1^4 e^{-x}\, dx$$

$$= -xe^{-x}\Big|_1^4 + \cancel{\int_1^4 e^{-x}\, dx} - \cancel{\int_1^4 e^{-x}\, dx}$$

$$\boxed{= -4e^{-4} + e^{-1}}$$

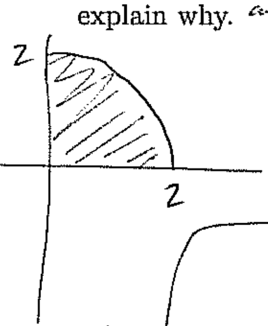
3. (3 points) Consider the iterated integral $\int_0^2 \int_0^{\sqrt{4-x^2}} \sin\sqrt{x^2+y^2}\, dydx$.

(a) Sketch the region of integration.

(b) Can you easily calculate the integral by hand as it is given? If so, explain why it is easy. If not, explain why it is hard.

(c) Would changing the order of integration or changing to polar coordinates help? If so, explain why. and set up new integral (do not integrate it!)

(a)



(b) No. Need a y in front in order to do integ. by substitution.

(c) Changing order of integration is no help.

Changing to polar could be good. $x^2+y^2=r^2$

$\Rightarrow \sin(r)$ as integrand

Limits would change to $0 \le r \le 2$

$0 \le \theta \le \frac{\pi}{2}$

$$\boxed{\text{get} \int_0^{\pi/2} \int_0^2 (\sin r)\, r\, dr\, d\theta}$$

(Need to do integ. by parts to solve)